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Local Discrete Symmetry in the Brane Neighborhood

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Abstract

With the ansatz that there exist local or global discrete symmetries in the special branes' neighborhoods, we can construct the extra dimension models with only zero modes, or the models which have large extra dimensions and arbitrarily heavy KK modes because there is no simple relation between the mass scales of extra dimensions and the masses of KK states. In addition, the bulk gauge symmetry and supersymmetry can be broken on the special branes for all the modes, and in the bulk for the zero modes by local and global discrete symmetries. To be explicit, we discuss the supersymmetric $SU(5)$ model on $M^4 \times S^1/Z_2$ in which there is a local Z'_2 symmetry in the special 3-brane neighborhood along the fifth dimension.

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1 Introduction

Large extra dimension scenarios with branes have been an very interesting subject for the past few years, where the gauge hierarchy problem can be solved because the physical volume of extra dimensions may be very large and the higher dimensional Planck scale might be low [1], or the metric for the extra dimensions has warp factor [2]. Naively, one might think the masses of KK states are $\sqrt{\sum_i n_i^2/R_i^2}$ where R_i is the radius of the i -th extra dimension. However, it is shown that this is not true if one considered the shape moduli, and it may be possible to maintain the ratio (hierarchy) between the higher dimensional Planck scale and 4-dimensional Planck scale while simultaneously making the KK states arbitrarily heavy [3]. So, a lot of experimental bounds on the theories with large extra dimensions are relaxed.

In this letter, we present another scenario where the higher dimensional Planck scale can be low while all the KK modes are projected out or the masses of KK states can be set arbitrarily heavy. Our ansatz is that there exist local or global discrete symmetries in the special branes' neighborhoods, which become the additional constraints on KK states. The KK states, which satisfy the local and global discrete symmetries, remain in the theory, while the KK states, which do not satisfy the local or global discrete symmetries, are projected out. Therefore, we can construct the theories with only zero modes for all the KK modes are projected out, or the theories which have large extra dimensions and arbitrarily heavy KK modes because there is no simple relation between the mass scales of extra dimensions and the masses of KK states. In addition, the bulk gauge symmetry and supersymmetry can be broken on the special branes for the zero and KK modes, and in the bulk for the zero modes by local and global discrete symmetries. As an example, we discuss the supersymmetric $SU(5)$ model on the space-time $M^4 \times S^1/Z_2$ in which there is a special 3-brane along the fifth dimension. In the neighborhood of special 3-brane, there exists a local Z'_2 symmetry, which is broken globally due to the presences of two boundary 3-branes. The bulk 4-dimensional $N = 2$ supersymmetry and $SU(5)$ gauge symmetry are broken down to the 4-dimensional $N = 1$ supersymmetry and $SU(3) \times SU(2) \times U(1)$ gauge symmetry on the special brane for all the states, and in the bulk for the zero modes. Moreover, all the KK states can be projected out or can be set arbitrarily heavy although the physical size of the fifth dimension can be large, even at millimeter range.

2 Local Discrete Symmetry in the Brane Neighborhood

We assume that in a $(4+n)$ -dimensional space-time manifold $M^4 \times M^n$ where M^4 is the 4-dimensional Minkowski space-time and M^n is the manifold for extra space dimensions, there exist some topological defects, or we call them branes for simplicity. The special branes, which we are interested in, have co-dimension one or more than

one. Assuming we have K special branes and using $I - th$ special brane as a representative, our ansatz is that in the open neighborhood $M^4 \times U_I$ ($U_I \subset M^n$) of the $I - th$ special brane, there is a local discrete symmetry², which forms a discrete group Γ_I where $I = 1, 2, \dots, K$. And the Lagrangian is invariant under the local discrete symmetries. Assume the local coordinates for extra dimensions in the $I - th$ special brane neighborhood are y^1, y^2, \dots, y^n , the action of any element $\gamma_i^I \in \Gamma_I$ on U_I can be expressed as

$$\gamma_i^I : (y^1, y^2, \dots, y^n) \in U_I \longrightarrow (\gamma_i^I y^1, \gamma_i^I y^2, \dots, \gamma_i^I y^n) \in U_I , \quad (1)$$

where the $I - th$ special brane position is the only fixed point, line, or hypersurface for the whole group Γ_I as long as the neighborhood is small enough.

The Lagrangian is invariant under the discrete symmetry in the neighborhood $M^4 \times U_I$ of the $I - th$ special brane, i. e., for any element $\gamma_i^I \in \Gamma_I$

$$\mathcal{L}(x^\mu, \gamma_i^I y^1, \gamma_i^I y^2, \dots, \gamma_i^I y^n) = \mathcal{L}(x^\mu, y^1, y^2, \dots, y^n) , \quad (2)$$

where $(y^1, y^2, \dots, y^n) \in U_I$. So, for a generic bulk multiplet Φ which fills a representation of the bulk gauge group G , we have

$$\Phi(x^\mu, \gamma_i^I y^1, \gamma_i^I y^2, \dots, \gamma_i^I y^n) = \eta_\Phi^I (R_{\gamma_i^I})^{l_\Phi} \Phi(x^\mu, y^1, y^2, \dots, y^n) (R_{\gamma_i^I}^{-1})^{m_\Phi} , \quad (3)$$

where η_Φ^I can be determined from the Lagrangian (up to ± 1 for the matter fields), l_Φ and m_Φ are the non-negative integers determined by the representation of Φ under the gauge group G . In general, η_Φ might be an element in the global symmetry for Lagrangian, and for the simple case, $\eta_\Phi = \pm 1$. Moreover, $R_{\gamma_i^I}$ is an element in G , and R_{Γ_I} is a discrete subgroup of G . We will choose $R_{\gamma_i^I}$ as the matrix representation for γ_i^I in the adjoint representation of the gauge group G . The consistent condition for $R_{\gamma_i^I}$ is

$$R_{\gamma_i^I} R_{\gamma_j^I} = R_{\gamma_i^I \gamma_j^I} , \quad \forall \gamma_i^I, \gamma_j^I \in \Gamma_I . \quad (4)$$

Mathematical speaking, the map $R : \Gamma_I \longrightarrow R_{\Gamma_I} \subset G$ is a homomorphism. Because the special branes are fixed under the local discrete symmetry transformations, the gauge group on the $I - th$ special brane is the subgroup of G which commutes with R_{Γ_I} . And for the zero modes, the bulk gauge group is broken down to the subgroup of G which commutes with all R_{Γ_I} , i. e., $R_{\Gamma_1}, R_{\Gamma_2}, \dots, R_{\Gamma_K}$, which is important if we wanted to discuss high rank GUT symmetry breaking. In addition, if the theory is supersymmetric, the special branes will preserve part of the bulk supersymmetry, and the bulk zero modes also preserve part of the supersymmetry, in other words, the supersymmetry can be broken on the special branes for all the modes, and in the bulk for zero modes.

²Global discrete symmetry is a “special” case of local discrete symmetry. The key difference is that, the space-time manifold can modulo the global discrete symmetry and become a quotient space-time manifold or orbifold.

In addition, we only have the KK states which satisfy the local and global discrete symmetries in the theories because the KK modes, which do not satisfy the local and global discrete symmetries, are projected out under our ansatz. Therefore, we can construct the theories with only zero modes because all the KK modes are projected out, or the theories which have large extra dimensions and arbitrarily heavy KK states for there is no simple relation between the mass scales of extra dimensions and the masses of KK states. To be explicit, we would like to discuss the KK mode expansions in an simple scenario.

Let us consider the 5-dimensional space-time which can be factorized into a product of the ordinary 4-dimensional Minkowski space-time M^4 and the orbifold S^1/Z_2 . The corresponding coordinates are x^μ , ($\mu = 0, 1, 2, 3$), $y \equiv x^5$, and the radius for the fifth dimension is R . The orbifold S^1/Z_2 is obtained by S^1 moduloing the equivalent class $y \sim -y$, and then, there are two fixed points: $y = 0$ and $y = \pi R$. Moreover, we assume that there is a special 3-brane along the fifth dimension, which is located at $0 < y = s < \pi R$. Our ansatz is that, there is a local Z'_2 symmetry in the neighborhood of special 3-brane, which becomes a global Z'_2 symmetry if $s = \pi R/2$. Mathematical speaking, we define $y' \equiv y - s$, and in the open neighborhood of sepcial 3-brane, we have the equivalent class $y' \sim -y'$, which can not be moduloed because it is not a global symmetry. Essentially speaking, we consider the 5-dimensional space-time $M^4 \times S^1$ with four special 3-branes which are located at $y = 0, \pi R, -s, s$, and the system has one non-equivalent global Z_2 symmetry and one non-equivalent local Z'_2 symmetry.

For a generic bulk multiplet $\Phi(x^\mu, y)$ which fills a representation of the gauge group G , we can define two parity operators P and P' for the Z_2 and Z'_2 symmetries, respectively

$$\Phi(x^\mu, y) \rightarrow \Phi(x^\mu, -y) = \eta_\Phi P^{l_\Phi} \Phi(x^\mu, y) (P^{-1})^{m_\Phi} , \quad (5)$$

$$\Phi(x^\mu, y') \rightarrow \Phi(x^\mu, -y') = \eta'_\Phi (P')^{l_\Phi} \Phi(x^\mu, y') (P'^{-1})^{m_\Phi} , \quad (6)$$

where $\eta_\Phi = \pm 1$, $\eta'_\Phi = \pm 1$. And in the discussions of next section, for simplicity, we assume that $\eta_\Phi = \eta'_\Phi$.

Denoting the field ϕ with $(P, P') = (\pm, \pm)$ by $\phi_{\pm\pm}$, we obtain that if $s/(\pi R - s)$ is not a rational number, then, we only have the zero modes because all the KK modes are projected out. And if $s/(\pi R - s)$ is a rational number, we can define relative prime integers p and q by

$$\frac{p}{q} = \frac{s}{\pi R - s} . \quad (7)$$

If p or q is even, the fields $\phi_{+-}(x^\mu, y)$ and $\phi_{-+}(x^\mu, y)$ are projected out. Only when p and q are odd, we have the full KK mode expansions

$$\phi_{++}(x^\mu, y) = \sum_{n=0}^{\infty} \frac{1}{\sqrt{2^{\delta_{n,0}} \pi R}} \phi_{++}^{(2n)}(x^\mu) \cos \frac{2ny}{r} , \quad (8)$$

$$\phi_{+-}(x^\mu, y) = \sum_{n=0}^{\infty} \frac{1}{\sqrt{\pi R}} \phi_{+-}^{(2n+1)}(x^\mu) \cos \frac{(2n+1)y}{r}, \quad (9)$$

$$\phi_{-+}(x^\mu, y) = \sum_{n=0}^{\infty} \frac{1}{\sqrt{\pi R}} \phi_{-+}^{(2n+1)}(x^\mu) \sin \frac{(2n+1)y}{r}, \quad (10)$$

$$\phi_{--}(x^\mu, y) = \sum_{n=0}^{\infty} \frac{1}{\sqrt{\pi R}} \phi_{--}^{(2n+2)}(x^\mu) \sin \frac{(2n+2)y}{r}, \quad (11)$$

where

$$r = \frac{2R}{p+q}, \quad (12)$$

and n is a non-negative integer. The 4-dimensional fields $\phi_{++}^{(2n)}$, $\phi_{+-}^{(2n+1)}$, $\phi_{-+}^{(2n+1)}$ and $\phi_{--}^{(2n+2)}$ acquire masses $2n/r$, $(2n+1)/r$, $(2n+1)/r$ and $(2n+2)/r$ upon the compactification. Zero modes are contained only in ϕ_{++} fields, thus, the matter content of massless sector is smaller than that of the full 5-dimensional multiplet. Moreover, only ϕ_{++} and ϕ_{+-} fields have non-zero values at $y=0$ and $y=\pi R$, and only ϕ_{++} and ϕ_{-+} fields have non-zero values at $y=s$. By the way, when $p=q=1$, i. e., $s=\pi R/2$, we obtain the previous KK mode expansions [4].

In short, from our simple scenario, we obtain that: (I) if $s/(\pi R - s)$ is not a rational number, we only have zero modes, i. e., $\phi_{++}^{(0)}(x^\mu)$; (II) if p or q is even, we only have the fields $\phi_{++}^{(2n)}(x^\mu)$ and $\phi_{--}^{(2n+2)}(x^\mu)$; (III) if p and q are odd, we will have the KK mode expansions for all the fields $\phi_{++}^{(2n)}(x^\mu)$, $\phi_{+-}^{(2n+1)}(x^\mu)$, $\phi_{-+}^{(2n+1)}(x^\mu)$ and $\phi_{--}^{(2n+2)}(x^\mu)$. In addition, because $0 < r \leq R$, the masses of KK states (n/r) can be set arbitrarily heavy if we choose suitable p and q , for instance, if $1/R$ is about TeV, $p=10^{13}-1$ and $q=10^{13}+1$, we obtain that $1/r$ is about 10^{16} GeV, which is the usual GUT scale. Therefore, there is no simple relation between the physical size of the fifth dimension and the mass scales of KK modes. Furthermore, the gauge symmetry and supersymmetry can be broken on the special 3-brane for all the modes, and in the bulk for the zero modes by choosing suitable P' .

3 Application to the Supersymmetric $SU(5)$ Model on $M^4 \times S^1/Z_2$

We would like to discuss the supersymmetric $SU(5)$ model on the space-time $M^4 \times S^1/Z_2$ with a special 3-brane which has local Z'_2 symmetry along the fifth dimension. We assume that the $SU(5)$ gauge fields and two 5-plet Higgs hypermultiplets in the bulk, and the Standard Model fermions can be on the 3-brane or in the bulk. The

$SU(5)$ gauge symmetry breaking mechanism is similar to those discussed by a lot of papers recently [5, 6].

As we know, the $N = 1$ supersymmetric theory in 5-dimension have 8 real supercharges, corresponding to $N = 2$ supersymmetry in 4-dimension. The vector multiplet physically contains a vector boson A_M where $M = 0, 1, 2, 3, 5$, two Weyl gauginos $\lambda_{1,2}$, and a real scalar σ . In terms of 4-dimensional $N = 1$ language, it contains a vector multiplet $V(A_\mu, \lambda_1)$ and a chiral multiplet $\Sigma((\sigma + iA_5)/\sqrt{2}, \lambda_2)$ which transform in the adjoint representation of $SU(5)$. And the 5-dimensional hypermultiplet physically has two complex scalars ϕ and ϕ^c , a Dirac fermion Ψ , and can be decomposed into two chiral multiplets $\Phi(\phi, \psi \equiv \Psi_R)$ and $\Phi^c(\phi^c, \psi^c \equiv \Psi_L)$, which transform as conjugate representations of each other under the gauge group. For instance, we have two Higgs chiral multiplets H_u and H_d , which transform as 5 and $\bar{5}$ under $SU(5)$ gauge symmetry, and their mirror H_u^c and H_d^c , which transform as $\bar{5}$ and 5 under $SU(5)$ gauge symmetry.

The general action for the $SU(5)$ gauge fields and their couplings to the bulk hypermultiplet Φ is [7]

$$\begin{aligned}
S = & \int d^5x \frac{1}{kg^2} \text{Tr} \left[\frac{1}{4} \int d^2\theta (W^\alpha W_\alpha + \text{H.C.}) \right. \\
& + \int d^4\theta \left((\sqrt{2}\partial_5 + \bar{\Sigma})e^{-V}(-\sqrt{2}\partial_5 + \Sigma)e^V + \partial_5 e^{-V} \partial_5 e^V \right) \Big] \\
& + \int d^5x \left[\int d^4\theta \left(\Phi^c e^V \bar{\Phi}^c + \bar{\Phi} e^{-V} \Phi \right) \right. \\
& \left. + \int d^2\theta \left(\Phi^c (\partial_5 - \frac{1}{\sqrt{2}}\Sigma) \Phi + \text{H.C.} \right) \right] . \tag{13}
\end{aligned}$$

Because the action is invariant under the parities P globally and P' in the special 3-brane neighborhood, we obtain that under the parity operator P , the vector multiplet transforms as

$$V(x^\mu, y) \rightarrow V(x^\mu, -y) = PV(x^\mu, y)P^{-1} , \tag{14}$$

$$\Sigma(x^\mu, y) \rightarrow \Sigma(x^\mu, -y) = -P\Sigma(x^\mu, y)P^{-1} , \tag{15}$$

if the hypermultiplet Φ is a 5 or $\bar{5}$ $SU(5)$ multiplet, we have

$$\Phi(x^\mu, y) \rightarrow \Phi(x^\mu, -y) = \eta_\Phi P\Phi(x^\mu, y) , \tag{16}$$

$$\Phi^c(x^\mu, y) \rightarrow \Phi^c(x^\mu, -y) = -\eta_\Phi P\Phi^c(x^\mu, y) , \tag{17}$$

and if the hypermultiplet Φ is a 10 or $\bar{10}$ $SU(5)$ multiplet, we have

$$\Phi(x^\mu, y) \rightarrow \Phi(x^\mu, -y) = \eta_\Phi P\Phi(x^\mu, y)P^{-1} , \tag{18}$$

Table 1: Parity assignment and masses ($n \geq 0$) of the fields in the $SU(5)$ gauge and Higgs multiplets. The indices F, T are for doublet and triplet, respectively.

(P, P')	field	mass
$(+, +)$	V_μ^a, H_u^F, H_d^F	$\frac{2n}{r}$
$(+, -)$	$V_\mu^{\hat{a}}, H_u^T, H_d^T$	$\frac{2n+1}{r}$
$(-, +)$	$\Sigma^{\hat{a}}, H_u^{cT}, H_d^{cT}$	$\frac{2n+1}{r}$
$(-, -)$	$\Sigma^a, H_u^{cF}, H_d^{cF}$	$\frac{2n+2}{r}$

$$\Phi^c(x^\mu, y) \rightarrow \Phi^c(x^\mu, -y) = -\eta_\Phi P \Phi^c(x^\mu, y) P^{-1}, \quad (19)$$

where $\eta_\Phi = \pm 1$. And the transformations of vector multiplet and hypermultiplet under the local Z'_2 symmetry P' are similar to those under the global Z_2 symmetry P .

We choose the following matrix representations for the global parity P and local parity P' which are expressed in the adjoint representation of $SU(5)$

$$P = \text{diag}(+1, +1, +1, +1, +1), \quad P' = \text{diag}(-1, -1, -1, +1, +1). \quad (20)$$

So, upon the local P' parity, the gauge generators T^A where $A=1, 2, \dots, 24$ for $SU(5)$ are separated into two sets: T^a are the gauge generators for the Standard Model gauge group, and $T^{\hat{a}}$ are the other broken gauge generators

$$P T^a P^{-1} = T^a, \quad P T^{\hat{a}} P^{-1} = T^{\hat{a}}, \quad (21)$$

$$P' T^a P'^{-1} = T^a, \quad P' T^{\hat{a}} P'^{-1} = -T^{\hat{a}}. \quad (22)$$

Choosing $\eta_{H_u} = +1$ and $\eta_{H_d} = +1$, we obtain the particle spectra, which are given in Table 1. And the gauge fields, Higgs fields, and gauge group on the 3-branes are given in Table 2. The bulk 4-dimensional $N = 2$ supersymmetry and $SU(5)$ gauge symmetry are broken down to the 4-dimensional $N = 1$ supersymmetry and $SU(3) \times SU(2) \times U(1)$ gauge symmetry on the special 3-brane for all the states, and in the bulk for the zero modes. Including the KK states, the gauge symmetry on the boundary 3-brane which is located at $y = 0$ or $y = \pi R$ is $SU(5)$. By the way, the 4-dimensional supersymmetry on the 3-branes at $y = 0, s, \pi R$ is $1/2$ of the bulk 4-dimensional supersymmetry or $N = 1$ due to the Z_2 or Z'_2 symmetry.

3.1 The Standard Model Fermions on the 3-Brane

If the Standard Model fermions were on the special 3-brane at $y = s$, the gauge symmetry is $SU(3) \times SU(2) \times U(1)$. Because the Higgs triplets are projected out on

Table 2: The gauge fields, Higgs fields and gauge group on the 3-branes which are located at $y = 0$, $y = s$, and $y = \pi R$.

Brane position	field	gauge group
$y = 0$	V_μ^A, H_u, H_d	$SU(5)$
$y = s$	$V_\mu^a, \Sigma^{\hat{a}}, H_u^F, H_d^F, H_u^{cT}, H_d^{cT}$	$SU(3) \times SU(2) \times U(1)$
$y = \pi R$	V_μ^A, H_u, H_d	$SU(5)$

the special brane, the Yukawa couplings are no more restricted than in the usual 4-dimensional Minimal Supersymmetric Standard Model, and there is enough flexibility to accommodate fermion masses and mixings. In short, the Yukawa terms in the superpotential of the 5-dimensional effective Lagrangian are

$$W_{\text{Yukawa}} = \int d^2\theta \delta(y-s) \sum_{I=1}^3 \sum_{J=1}^3 \left(h_{IJ}^u Q^I H_u^F \bar{U}^J + h_{IJ}^d Q^I H_d^F \bar{D}^J + h_{IJ}^l L^I H_d^F \bar{E}^J \right), \quad (23)$$

where the Q , U , D , L , E denote the quark $SU(2)_L$ doublet, right-handed up-type quark, right-handed down-type quark, lepton/neutrino $SU(2)_L$ doublet, and right-handed lepton, respectively. So, we avoid the wrong prediction of the first and second generation fermion mass ratios, m_d/m_e and m_s/m_μ in the usual 4-dimensional $SU(5)$ model. And if the Standard Model fermions only preserve the $SU(3) \times SU(2) \times U(1)$ gauge symmetry, there are no proton decay problem at all. However, we can not explain the charge quantization. By the way, we can put two Higgs doublets on the special 3-brane instead of putting two Higgs 5-plets in the bulk.

If the Standard Model fermions were on the boundary 3-brane at $y = 0$ or $y = \pi R$, the gauge symmetry is $SU(5)$. However, we can not define the Yukawa terms in the superpotential properly unless $s = \pi R/2$. So, we will not discuss it here.

3.2 The Standard Model Fermions in the Bulk

Now, we consider the scenario where the Standard Model fermions are in the Bulk. We will double the generations in the bulk due to the P' projections, i. e., we will have $T^I + \bar{F}^J + T^{cI} + \bar{F}^{cJ}$ and $T'^I + \bar{F}'^I + T'^{cI} + \bar{F}'^{cI}$ where $I = 1, 2, 3$, c denotes the charge conjugation, and $\eta_{T^I} = \eta_{\bar{F}^J} = 1$, $\eta_{T'^I} = \eta_{\bar{F}'^J} = -1$. And each generation in the Standard Model comes from the zero modes of two generations. Moreover, the particle spectra for the bulk fermions are given in Table 3.

Because the fermions and Higgs fields are in the bulk, we need to define the

Table 3: Parity assignment and masses ($n \geq 0$) of the bulk fermions in the $10 + \bar{5}$ $SU(5)$ multiplets.

(P, P')	field	mass
$(+, +)$	$T_U^I, T_E^I, \bar{F}_L^I, T_Q'^I, \bar{F}_D'^I$	$\frac{2n}{r}$
$(+, -)$	$T_Q^I, \bar{F}_D^I, T_U'^I, T_E'^I, \bar{F}_L'^I$	$\frac{2n+1}{r}$
$(-, +)$	$T_Q^{cI}, \bar{F}_D^{cI}, T_U^{cI}, T_E^{cI}, \bar{F}_L^{cI}$	$\frac{2n+1}{r}$
$(-, -)$	$T_U^{cI}, T_E^{cI}, \bar{F}_L^{cI}, T_Q^{cI}, \bar{F}_D^{cI}$	$\frac{2n+2}{r}$

bulk Yukawa terms. The relevant bulk Yukawa terms in the superpotential are

$$W_{\text{Yukawa}} = \int d^2\theta \sum_{I=1}^3 \sum_{J=1}^3 \left((\theta(y) - \theta(-y)) h_{IJ}^u T'^I T^J H_u + h_{IJ}^d T'^I \bar{F}^J H_d + h_{IJ}^l T^I \bar{F}^J H_d \right), \quad (24)$$

where $\theta(x)$ is the step function defined as $\theta(y) = 1$ for $y \geq 0$, and $\theta(y) = 0$ for $y < 0$. And for the zero modes, the Yukawa terms in the superpotential become

$$W_{\text{Yukawa}} = \int d^2\theta \sum_{I=1}^3 \sum_{J=1}^3 \left(h_{IJ}^u T_Q'^I T_U^J H_u^F + h_{IJ}^d T_Q'^I \bar{F}_D^J H_d^F + h_{IJ}^l T_E^I \bar{F}_L^J H_d^F \right). \quad (25)$$

In this scenario, we avoid the wrong $SU(5)$ prediction of the first and second generation fermion mass ratios, and have charge quantization. In addition, the tree-level proton decays by exchange X , Y and Higgs triplets are absent because D and L , Q and U/E come from different hypermultiplets.

3.3 Phenomenology Comments

In our model, the physical radius of the fifth dimension is R , while the masses of KK states is n/r . Because $0 < r \leq R$, we can have large extra dimension and arbitrarily heavy KK states. For example, $1/R$ might be at TeV scale and the masses of KK states can be at 10^{16} GeV. Especially, when we consider the non-supersymmetric $SU(5)$ model on the space-time $M^4 \times S^1/Z_2$, we might solve the gauge hierarchy problem by having large extra dimension, and avoid the proton decay by giving X , Y , and Higgs triplet masses about 10^{16} GeV because they do not have zero modes. Furthermore, we can push the physical size of the fifth dimension to the millimeter range, while the masses of KK states are still at TeV scale.

In addition, if one considered the Scherk-Schwartz mechanism of supersymmetry breaking, the soft mass scale and the possible μ term might be at order of $1/R$,

which can be at TeV scale naturally, unlike in previous models, where $1/R$ is the GUT scale [5, 6].

4 Discussion and Conclusion

In section 3, one can also choose the matrix representations for the global parity P and local parity P' as

$$P = \text{diag}(-1, -1, -1, +1, +1) , \quad P' = \text{diag}(+1, +1, +1, +1, +1) . \quad (26)$$

So, the gauge group on the boundary 3-brane at $y = 0$ or $y = \pi R$ is $SU(3) \times SU(2) \times U(1)$, and the gauge group on the special 3-brane at $y = s$ is $SU(5)$. The discussions for the Standard Model fermions on the boundary 3-brane or in the bulk are similar to those in above section. So, let us discuss the case where the Standard Model fermions are on the special 3-brane with $SU(5)$ gauge symmetry. As we know, the transformation properties of the quark and lepton superfields under the Z_2 symmetry or P parity are determined by the requirement that any operators on the special 3-brane must transform $SU(5)$ covariantly under Z_2 symmetry. Because the kinetic terms for the 10 T^I and $\bar{5}$ \bar{F}^J $SU(5)$ multiplets must transform covariantly under Z_2 symmetry where $I, J = 1, 2, 3$, there are only four possibilities for the assignment of the P quantum numbers: (I) $P(Q, U, D, L, E) = \pm(+, -, -, +, -)$ or (II) $P(Q, U, D, L, E) = \pm(-, +, -, +, +)$. And then, we can obtain the transformation properties of the Yukawa couplings $[T^I T^J H_u]_{\theta^2}$ and $[T^I \bar{F}^J H_d]_{\theta^2}$ under the P parity: for the case (I) $P(T^I T^J H_u) = P(T^I \bar{F}^J H_d) = -$; and for the case (II) $P(T^I T^J H_u) = -P(T^I \bar{F}^J H_d) = -$. Therefore, the Z_2 invariant Yukawa interactions in the superpotential are

$$\begin{aligned} W_{\text{Yukawa}} = & \int d^2\theta \sum_{I=1}^3 \sum_{J=1}^3 \left[\frac{1}{2} \{ \delta(y-s) - \delta(y+s) \} \sqrt{2\pi R} h_{IJ}^u T^I T^J H_u \right. \\ & \left. + \frac{1}{2} \{ \delta(y-s) \mp \delta(y+s) \} \sqrt{2\pi R} h_d^{IJ} T^I \bar{F}^J H_d \right] , \end{aligned} \quad (27)$$

where \mp takes $-$ and $+$ in the case of (I) and (II), respectively. However, we have the wrong $SU(5)$ prediction of the first and second generation fermion mass ratios, which may be solved by adding bulk hypermultiplets which transform as $5 + \bar{5}$ [6].

Furthermore, one can consider the models on the space-time $M^4 \times S^1$ with two local Z_2 symmetries, and the models on the space-time $M^4 \times T^2$ with local discrete symmetries, like $(Z_2)^3$, $(Z_2)^4$, Z_3 , or Z_6 , etc. Those generalizations will be discussed elsewhere [8].

In short, with the ansatz that there exist local or global discrete symmetries in the special branes' neighborhoods, we can construct the extra dimension models with only zero modes, or the models which have large extra dimensions and arbitrarily heavy KK modes because there is no simple relation between the mass scales of extra

dimensions and the masses of KK states. In addition, the bulk gauge symmetry and supersymmetry can be broken on the special branes for all the modes, and in the bulk for the zero modes by local and global discrete symmetries. To be explicit, we discuss the supersymmetric $SU(5)$ model on $M^4 \times S^1/Z_2$ in which there is a local Z'_2 symmetry in the special 3-brane neighborhood along the fifth dimension.

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